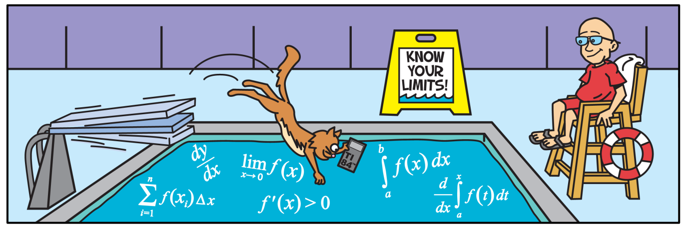
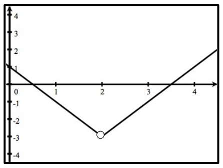
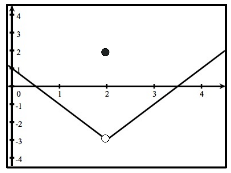
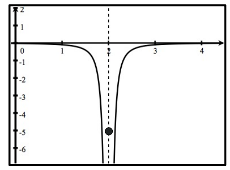
Diving In
The AP Calculus Exam



1) Limits

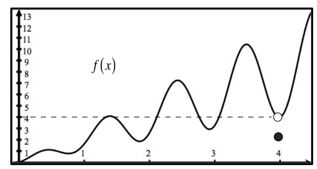
1. For which of the following functions f does $\lim_{x\to 2} f(x)$ exist?

I)  II)  III) 

A) I only B) II only C) III only D) I and II only

> D. Whether $f(2)$ exists doesn't matter. We are only interested in whether $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$.
> This occurs in I and II. In III, $\lim_{x\to 2^-} f(x) = -\infty$ and $\lim_{x\to 2^+} f(x) = -\infty$. These limits do not exist.

2. Using the graph of the function f to the right, which of the following is correct?



I. $\lim_{x\to 4} f(x) = 4$ II. $\lim_{x\to\infty} f(x) = \infty$ III. $\lim_{x\to\infty} f(x)$ does not exist

A) I only B) I and II only C) I and III only D) II and III only

> C. The value of $f(4)$ doesn't matter. Since $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = 4$, $\lim_{x\to 4} f(x) = 4$.
> The graph is oscillating as we go further to the right and doesn't approach a limit.
> But $\lim_{x\to\infty} f(x) \neq \infty$ as the function is not steadily increasing.

3. Find $\lim_{x\to\infty} \dfrac{2x}{\sqrt{x^2+x+100}} + \lim_{x\to -\infty} \dfrac{2x}{\sqrt{x^2+x+100}}$

A) 0 B) 2 C) 4 D) does not exist

> A. This translates to $\lim_{x\to\infty} \dfrac{2x}{+x} + \lim_{x\to -\infty} \dfrac{2x}{+x} = 2 + (-2) = 0$.

4. Find $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

A) 0 B) $\dfrac{1}{2}$ C) $\dfrac{1}{4}$ D) $\dfrac{1}{6}$

> C. $\lim_{x\to 4} \left(\dfrac{\sqrt{x}-2}{x-4}\right)\left(\dfrac{\sqrt{x}+2}{\sqrt{x}+2}\right) = \lim_{x\to 4} \left[\dfrac{x-4}{(x-4)(\sqrt{x}+2)}\right] = \lim_{x\to 4} \dfrac{1}{\sqrt{x}+2} = \dfrac{1}{4}$

5. Let $f(x)=\dfrac{ax^2+ax+b}{x^2-1}$.

a. If $\lim_{x\to 3} f(x)=\dfrac{5}{2}$, find b in terms of a. **(2)**

$$\frac{9a+3a+b}{9-1}=\frac{5}{2}$$
$$\frac{12a+b}{8}=\frac{5}{2}$$
$$24a+2b=40$$
$$b=\frac{40-24a}{2}=20-12a$$

1 pt for $\dfrac{12a+b}{8}=\dfrac{5}{2}$

1 pt for $b=20-12a$

b. If $\lim_{x\to -3} f(x)=1$, find the values of a and b. **(3)**

$$\frac{9a-3a+b}{9-1}=1$$
$$6a+b=8$$
$$6a+20-12a=8$$
$$6a=12$$
$$a=2$$

$$b=20-12(2)$$
$$b=20-24$$
$$b=-4$$

1 pt for $6a+b=8$

1 pt for a

1 pt for b

c. Find $\lim_{x\to 1}\dfrac{2x^2+2x-4}{x^2-1}-\lim_{x\to\infty}\dfrac{2x^2+2x-4}{x^2-1}$. **(3)**

$$\lim_{x\to 1}\frac{2x^2+2x-4}{x^2-1}$$
$$\lim_{x\to 1}\frac{2(x^2+x-2)}{x^2-1}$$
$$\lim_{x\to 1}\frac{2(x+2)(x-1)}{(x+1)(x-1)}$$
$$\lim_{x\to 1}\frac{2(x+2)}{x+1}=\frac{2(3)}{2}=3$$

$$\lim_{x\to\infty}\frac{2x^2+2x-4}{x^2-1}=2$$

$$\lim_{x\to 1}\frac{2x^2+2x-4}{x^2-1}-\lim_{x\to\infty}\frac{2x^2+2x-4}{x^2-1}=3-2=1$$

1 pt for $\lim_{x\to 1}\dfrac{2(x+2)(x-1)}{(x+1)(x-1)}$

1 pt for $\lim_{x\to 1} f(x)=3$

1 pt for $\lim_{x\to\infty} f(x)=2$