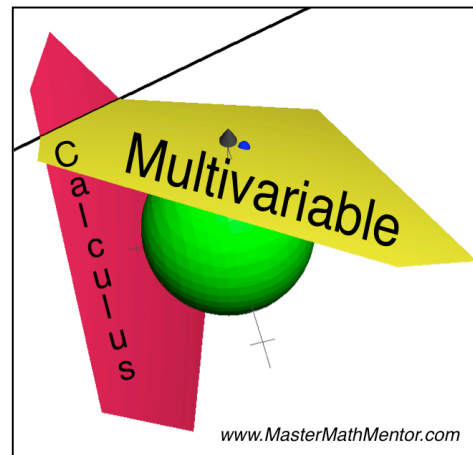


Multivariable Calculus

Need to start up a new Multivariable Calculus Course? Here's what you've been looking for.

Evolution of the Multivariable Calculus Materials

I retired from public school teaching in June 2007. Several parents whose kids were taking BC calculus with me in their Junior year were concerned that there was no math course for them as Seniors. They worked with the school to try and set up a dual credit Multivariable Calculus course with one of the neighboring colleges. Although I technically had nothing to do with it, I was asked to sit in the meetings as a consultant as I knew calculus and I also knew the students.



A local community college that has some experience with dual credit courses was chosen to deliver the course and they then worked to identify an instructor who would be willing to travel to the high school. It was difficult to find such a person and the community college people asked me if I would consider teaching the course.

I was quite reticent at first. I hadn't seen this material since my college years, 38 years ago. I knew it would take a lot of work to create a course. I had access to previous exams and finals from the community college, but it was clear to me that I would have to teach myself everything from scratch. Other than the basic vector work, I remembered very little. Still, I like challenges and I decided to teach the course and would work during the summer to set up a course. There were 10 students scheduled for the course.

The course was paid for by the students but was taught at the high school for the convenience of the students. It would be a 4-credit course. We needed 56 hours of class time. It was designed so that we would meet 3 days a week and the students would work together in a proctored situation (not by me) or in the library on homework problems. If you are interested in how the course was set up, please contact me.

For the course curriculum, I tried to follow that of the community college. I wanted to use the recommended textbook and technology. The text that the community college used was Calculus (8th edition) by Larson, Hostetler, and Edwards. The high school had a number of copies of the 5th edition which is substantially the same material, and we were able to use them, saving the students over \$100. However, students were required to have a TI-89 or TI-Inspire CAS and I adhered to that policy. And while we did not use it that much early in the course, it is invaluable in the section on iterated integrals. We also used other technology that I will mention later.

For years, I have been using the calculus manuals that I have developed and a number of you now use. Now I had to start from scratch. I started to fill up yellow legal pads with notes. I did not complete the course by the start of the school year. I usually was a month ahead of the students and it took a great deal of time to understand and develop the material. That is important – a lot of this work is not obvious and it took a good amount of time to understand terms and definitions. And after I developed sample problems (trying not to use the sample problems in the text), I then had to determine how to explain it. I dislike teaching from textbooks and find that students dislike learning from them, no matter how well written they are.

I have now taught the course a total of three years. My legal pads are falling apart and I started to develop packets similar to what I use in A.P. Calculus. This year, I decided to bite the bullet and put all my notes in a form that I could give to students and make available on my website.

The Multivariable Calculus Manuals

The Multivariable Calculus material has many similarities to my A.P. Calculus manuals and some important differences.

- Like the manuals, there is a student version and a solution version and there is a one-to-one relationship between pages. That is, problems in the student version on page 4, for example will have solutions on page 4.
- Like the manuals, there is room for students to write solutions to sample problems in the student manual.
- Like the manual, the solution manual will show exactly how to solve the problem. It is in PDF format so it can be projected onto a big screen. All answers are boxed.
- Like the manual, there is a Classwork section. These are problems that I show in class. I have more problems than I normally have because of their complexity. Some of them are quite involved.
- Like the manual, I have all important formulas boxed so a student's eyes are easily caught by them.
- Unlike the manual, there is no Homework section. The reason for this is the fact that this is a true college course and it will be up to the students as to how much work they do to prepare for exams. I do include a set of recommended problems to do in the Calculus textbook (using odd problems that have answers in the back).
- Unlike the manual, all references to graphics are boxed. I will mention graphics below.

Course Syllabus (following Larson Hostetler Calculus)

Chapter 10: Conics (Optional)

At the high school I taught at, supposedly we taught the Conic Sections. But it was always at the end of the school year and the material was barely covered. Students rarely remember it. When we get to quadric sections in chapter 11, it is important that students have a working knowledge of conics so I included it (some taken from my Precalculus manual) and started the year with it. I go through it quickly but spend some time on the rarely-taught rotation of axes. This gives students a chance to understand the power of their TI-89.

1. Inclination of a line	5. Circles	9. Determining Conic Type
2. Angle between lines	6. Parabolas	10. Rotation of Axes
3. Distance-pt and line	7. Ellipses	11. Homework
4. Conics	8. Hyperbolas	

Chapter 11: Vectors

Students in BC Calculus have had vectors. I review them a bit but quickly move onto 3-D vectors and the whole issue of 3-space. A lot of the material in this chapter and other chapters are taught by enlarging concepts taught in the 2-D world. The interesting quadric surfaces are taught in this chapter and technology is needed to illustrate. Again, read the section on graphics below.

1. 2-D Vectors	4. Projections	7. Planes in Space
2. 3-D Vectors	5. Cross Products	8. Surfaces in Space
3. Dot Products	6. Lines in Space	9. Other Coordinate Systems

Chapter 12: Vector-Valued Functions

Again, this was slightly touched upon in Calculus of a Single Variable but we quickly use analogies to move it to 3-space and beyond. The goal of this relatively short chapter is to move to the concept of curvature.

1. Vector-Valued Functions	4. Tangent Vectors & Normal Vectors
2. Calculus and Vector-Valued Functions	5. Arc Length & Arc Length Function
3. Position, Velocity, Speed and Acceleration	6. Curvature

Chapter 13: Functions of Several Variables

This is a long chapter and will probably be the longest students will go without a major math test. It reviews a lot of concepts from AP Calculus and extends them to 3 dimensions and greater. The basic concepts of partial derivatives and the relationship to slope in the 3rd dimension is investigated. Theoretical and applied problems in relative extrema are explored with many examples.

1. Functions of Several Variables	6. Directional Derivatives
2. Limits and Continuity	7. Tangent Planes and Normal Lines
3. Partial Derivatives	8. Absolute and Relative Extrema
4. Differentiability and Differentials	9. Applied Problems in Optimization
5. The Chain Rule	10. LaGrange Multipliers

Chapter 14: Multiple Integration

As in AP Calculus, we follow the exploration of derivatives and their applications to definite integration and its natural application, area and volume. Volume was attacked in AP calculus only by rotating a curve around an axis or line. We now look at volume in a much more natural way, using iterated integrals. In this section, we use TI-89 technology quite a bit as double or triple integrals are cumbersome to take by paper and pencil techniques.

- | | |
|-------------------------------|----------------------------------|
| 1. Iterated Integrals | 4. Volume in Polar Form |
| 2. Area of Plane Region | 5. Definition of Triple Integral |
| 3. Volume in Rectangular Form | 6. Center of Mass |

Chapter 15: Vector Analysis

We do not cover the entire chapter as this course is covering only the material in the community college course that I was teaching. Vector fields and especially conservative vector fields are explored. The interesting concept of line integrals is explored and how Green's theorem allows us to compute the value of a line integral over a certain path. Stoke's Theorem (sometimes in Multivariable courses) is not included.

- | |
|--------------------|
| 1. Vector Fields |
| 2. Line Integrals |
| 3. Green's Theorem |

So, it's only 5 chapters (6 if you include the conics), but there is a lot of material here. And although there are similarities between AP Calculus as you know it, there is much new here and it is essential to understand how important vectors are.

Technology:

In AP Calculus, technology is important but not vital, as evidenced by the AP testers now switching from three questions using calculators to two. In terms of graphics, technology is not vital as we can usually sketch curves fairly easily by hand.

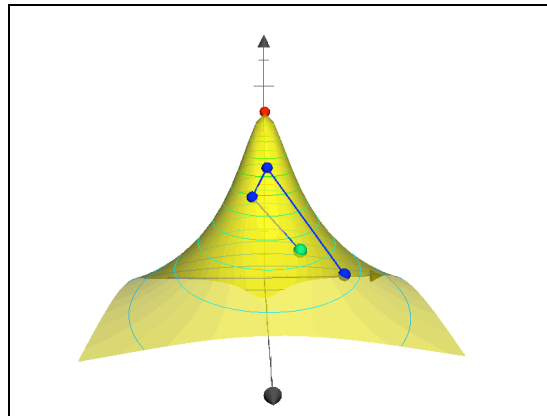
I have found that in Multivariable Calculus, calculator technology is really only important when multiple integrals have to be calculated and the TI-89 or equivalent is terrific for that. But more important is drawing these complex 3-D figures and lines and planes in space.

It is difficult to draw in 3-D and other than basic points and some planes, I rarely try. It is simple to draw a parabola but a paraboloid is a different animal. So if you want to illustrate a problem like finding the tangent plane to a surface at a point (analogous to finding a tangent line to a curve at a point), you are going to need some software.

I have been a Macintosh user for years and am very fond of the program, Grapher, that is part of the computer's operating system and found in the Utilities folder. Grapher isn't particularly well documented, but it can do some amazing things and quickly.

So, for people who have Macintosh computers, I have included 151 Grapher files for all of sample problems that warrant graphic descriptions. Once the file is loaded, users can see the shapes from any viewpoint by rotating them. Students have told me that this is the best aspect of the course, allowing many problems to "come alive."

For people not lucky enough to have a Macintosh, I felt that I had to help them as well. So I used the Grapher program to take static pictures of these graphics. They will not rotate, but in many cases, I show them from several angles. This results in 269 graphic files that are carefully organized by chapter and general topic. For instance, below is a graphic that represents a problem where students have to trace the route of mountain climbers attempting to reach the summit, encountering bad weather and trying to descend to base camp. For those with Macintosh computers, this graphic can be viewed from any angle.



There are also some very good web programs that allow users to graph 3-D shapes and even allow implicit input of equations. The downside is speed and the need to input the equations yourself. One of them is sponsored by the Lawrenceville School called Equation Plotter:

<http://mathplotter.lawrenceville.org/mathplotter/mathPage/index.htm>.

In my materials, all of the equations have been pre-input and scaled correctly so you can concentrate on teaching.

Exams:

I tried to create a course that, although taught at the high school, resembled a college course in every way. Testing was an area to which I tried to be consistent to college courses. Rather than the “test every 2 weeks” approach that I took in AP Calculus with some smaller quizzes, I had major exams at the end of each chapter, except the last. That gave me 5 exams (as I started with the Conics chapter) and a final exam. The final exam counts as a double exam. This creates a situation where students really have to study for the exams as each counts about 14% of the final grade.

There are a lot of formulas in this course, many more than in AP Calculus. My feelings are that although I can ask the students to memorize them, they will forget them the moment they walk out of the exam. So I allow the students to bring into the exam a 4 by 6 card with any formulas or theorems written on it. This helps them with studying and deciding what is important.

I also found that it was impossible to adequately test these long chapters in the standard 45 minutes. With the longer class periods of block scheduling, it could be done. So for each topic, I created two exams. Each covered all topics in the exam. I gave these exams on two contiguous days, and they lasted an hour, running into the next period. The school administration was good on allowing a bit of extra time. It was tough knowing how much time to leave for problems.

In chapter 14, one of the exams is designated “no calculator” as I want to see if the students could take double or triple integrals by hand. Applications were on the 2nd day exam where calculators were allowed.

There is no testing of chapter 15 topics specifically. They are covered in the final exam and represent about 20% of the final. This 3-hour exam is comprehensive and I allow the students a note card for each chapter. The final counts twice as much as any individual chapter test.

This last year, a local college contracted me to teach Multivariable Calculus. The students enrolled in this course were notably weaker than students I have taught over the past few years as they were not products of A.P. programs. I had to water down the course somewhat and thus create exams that were somewhat easier. I did so and for chapters 11, 13, and 14, I have included those exams, called Summary Exams, as well. Chapters 11 and 13 also have a practice exam.

So for all chapters I have Comprehensive exams and for some chapters, I have Summary Exams.

Samples:

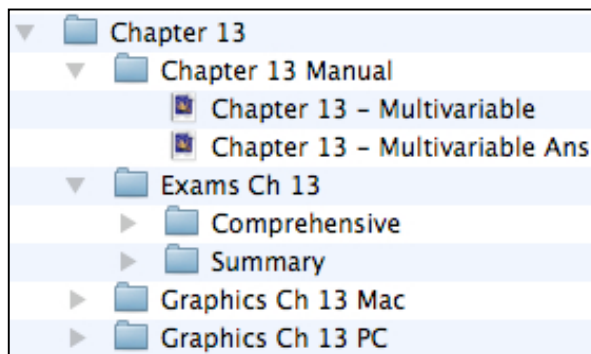
I have made several files available for you to give you a feel for the material.

1. Sample Manual: I have included the section on Directional Derivatives from Chapter 13.
2. Sample Manual Solutions: I have included the solutions for the same material on Directional Derivatives as above. When you look at it, you will see boxed terms like this: Gradient 1 That refers to the graphics that accompany the problem that appears in both the Mac graphic folder and the PC graphic folder.
3. Sample Exam Solutions: I have included the solutions to the entire chapter 13 second day exam.
4. Sample Graphic for Grapher program: This is for Macintosh computer users only.

What is Included in the Multivariable Calculus Materials:

Chapter	10	11	12	13	14	15	
Topic	Conics	Vectors	Vector-Valued Functions	Functions of Several Variables	Multiple Integration	Vector Analysis	Final
Student Manual	21 pages	30 pages	21 pages	42 pages	19 pages	16 pages	9 pages
Solution Manual	21 pages	30 pages	21 pages	42 pages	19 pages	16 pages	9 pages
Comprehensive Exam	1 day exam	2 day exams	2 day exams	2 day exams	2 day exams	2 day exams	3 hour exam
Comprehensive Exam Soln's	1 day exam	2 day exams	2 day exams	2 day exams	2 day exams	2 day exams	3 hour exam
Summary Exam	No	Yes – also Practice exam	No	Yes – also Practice exam	Take home type	No	No
Summary Exam Soln's	No	Yes	No	Yes	Yes	No	No
Mac Grapher Graphics	2 items	43 items	23 items	46 items	33 items	4 items	
PC Static Graphics	11 items	83 items	24 items	110 items	35 items	5 items	

With all of this material, it is important that it be organized. To the right is the typical organization of a folder on the CD so that everything can easily be found. You also get a page describing the final exam as well as suggested homework problems from the textbook.



Cost:

Unlike my AP Calculus material, there is little sense in making available downloads of the student manuals. Teachers who are going to teach this course are going to need the solution manuals as well.

So, all of this is available in just one method, on a CD. There is too much material for downloading – a total of 539 items. The cost of the Multivariable Calculus Course on CD is **\$199.95** plus \$2.50 shipping.

If you, like me, are being asked to start up a Multivariable Calculus course and you haven't seen this material in a number of years, this CD is everything you need to develop the course and will walk you through it step by step. You can also call on me for any support you need as you phase the course in.

Questions? [Click here to email us at Team@MasterMathMentor.com](mailto:Team@MasterMathMentor.com)