

Solutions – Jeopardy Sample

Go Straight 100

What is C?

$$s(t) = \frac{1}{2}e^{2t} - t^2 + C$$

$$s(0) = \frac{1}{2} + C = 0 \Rightarrow C = -\frac{1}{2}$$

$$s(t) = \frac{1}{2}e^{2t} - t^2 - \frac{1}{2}$$

$$s(1) = \frac{e^2 - 3}{2}$$

Go Straight 200

What is B?

Say at $t = 1$, the particle is at $y = 0$

At $t = 2$, the particle is at $y = -9$

At $t = 3$, the particle is at $y = -8$

At $t = 5$, the particle is at $y = 4$ so it passed $y = 0$

At $t = 6$, the particle is at $y = -4$ so it passed $y = 0$

Go Straight 300

What is D?

$$v\left(\frac{1}{4}\right) = \frac{-\sin\left(\frac{\pi}{2}\right)}{\frac{1}{4}} = -4 : \text{Particle moving left}$$

$$a(t) = \frac{-2t\pi \cos(2t\pi) + \sin(2t\pi)}{t^2}$$

$$a\left(\frac{1}{4}\right) = \frac{-2t\pi \cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)}{\frac{1}{16}} = 16 : \text{Acceleration to the right}$$

So particle is slowing down

Go Straight 400

What is 335 miles?

$$\int_0^{12} |v(t)| dt \approx 2 \frac{|v(0)| + |v(2)|}{2} + 3 \frac{|v(2)| + |v(5)|}{2} + 4 \frac{|v(5)| + |v(9)|}{2} + 3 \frac{|v(9)| + |v(12)|}{2}$$
$$\int_0^{12} |v(t)| dt \approx 2 \frac{20 + 50}{2} + 3 \frac{50 + 20}{2} + 4 \frac{20 + 30}{2} + 3 \frac{30 + 10}{2}$$
$$\int_0^{12} |v(t)| dt \approx 70 + 105 + 100 + 60 = 335 \text{ miles}$$

Go Straight 500

What is E?

$$v(t) = e^{-t}(\cos t + \sin t) - e^{-t}(\sin t - \cos t)$$
$$= 2e^{-t} \cos t = 0$$

$$t = \frac{\pi}{2}, \frac{3\pi}{2}$$

t	$x(t)$
0	$e^0(-1) = -1$ (furthest left)
$\frac{\pi}{2}$	$e^{-\pi/2}(1) = e^{-\pi/2} = .208$ (furthest right)
$\frac{3\pi}{2}$	$e^{-3\pi/2}(-1) = -e^{-3\pi/2} = -.009$
2π	$e^{-2\pi}(1) = e^{-2\pi} = .002$

$$\text{Distance} = .208 - (-1) = 1.208$$

Things Change 100

What is B?

$$V = \pi r^2 h$$

$$V = 400\pi h$$

$$\frac{dV}{dt} = 400\pi \frac{dh}{dt}$$

$$1000(.133) = 400\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = .106 \frac{\text{ft}}{\text{min}}$$

The 8 feet doesn't enter into the problem as the height increases at a constant rate since the sides of the cylinder are vertical.

Things Change 200

What is 490?

People entering :

$$2\left(\frac{380 + 450}{2}\right) + 1\left(\frac{450 + 200}{2}\right) + 2\left(\frac{200 + 30}{2}\right) = 1385$$

People leaving :

$$2\left(\frac{100 + 80}{2}\right) + 1\left(\frac{80 + 420}{2}\right) + 2\left(\frac{420 + 620}{2}\right) = 1470$$

$$575 + 1385 - 1470 = 490$$

Things Change 300

What is A?

$$x = 4, \frac{dx}{dt} = -40, y = 3, \frac{dy}{dt} = 20$$

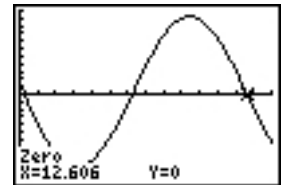
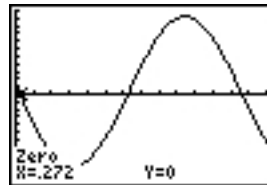
$$d = \sqrt{x^2 + y^2}$$

$$\frac{dd}{dt} = \frac{1}{2\sqrt{x^2 + y^2}} \left(2x \frac{dx}{dt} + 2y \frac{dy}{dt} \right) = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{\sqrt{x^2 + y^2}}$$

$$\frac{dd}{dt} = \frac{4(-40) + 3(20)}{\sqrt{40^2 + 30^2}} = \frac{-100}{50} = -2$$

Things Change 400

What is E?



$$f'(t) = (2t)^{1/2} + 20 \cos(.5t) - 15$$

$$f''(t) = \frac{1}{(2t)^{1/2}} - 10 \sin(.5t) = 0$$

The graph of $f''(t)$ shows that there are two possibilities for the maximum rate of change at $t = .272$ and $t = 12.606$. We also must examine the endpoints.

$$f'(0) = 5$$

$$f'(.272) = 5.553$$

$$f'(12.606) = 10.017$$

$$f'(14) = 5.370$$

The maximum occurs when $t = 12.606$ which corresponds to year 2002.

Things Change 500

What is B?

$$V = s^3$$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt} = ks$$

$$\int 3s \, ds = k \, dt$$

$$\frac{3s^2}{2} = kt + C$$

$$3s^2 = 2kt + C \Rightarrow 3(4) = C \Rightarrow C = 12$$

$$3s^2 = 2kt + 12 \Rightarrow 3(16) = 6k + 12 \Rightarrow k = 6$$

$$3s^2 = 12t + 12 \Rightarrow s^2 = 4(t + 1)$$

$$s = 2\sqrt{t + 1}$$

$$s(8) = 6$$

Hit the Slopes – 100

What is E?

$$y = \sin^{-1}(e - x)$$

$$y' = \frac{-1}{\sqrt{1 - (e - x)^2}} \Rightarrow y'(e) = \boxed{y'(e) = -1}$$

$$y = \frac{x^3 + 1}{x + 1} = \frac{(x + 1)(x^2 - x + 1)}{x + 1} = x^2 - x + 1$$

$$y' = 2x - 1 \Rightarrow \boxed{y'(e) = 2e - 1}$$

$$y = \ln\left(\frac{2x + e}{x + 2e}\right)^2 = 2[\ln(2x + e) - \ln(x + 2e)]$$

$$y' = 2\left[\frac{2}{2x + e} - \frac{1}{x + 2e}\right] \Rightarrow \boxed{y'(e) = 2\left[\frac{2}{3e} - \frac{1}{3e}\right] = \frac{2}{3e}}$$

Hit the Slopes – 200

What is B?

At $x = 0$, $y' \approx 1$, eliminating A. For larger values of x the slope is negative and C, D, E would all have positive slopes.

Hit the Slopes – 300

What is D?

$$y = f(x)g(x) \Rightarrow y' = f(x)g'(x) + g(x)f'(x)$$

$$y'(1) = f(1)g'(1) + g(1)f'(1) = 4(-2) + 2(3) = \boxed{-2}$$

$$y = \frac{1}{f^2(x)} = f^{-2}(x) \Rightarrow y' = \frac{-2}{f^3(x)}$$

$$y'(1) = \frac{-2}{4^3} = \boxed{\frac{-1}{32}}$$

$$y = \sqrt{f(x) + g(x)} \Rightarrow y' = \frac{f'(x) + g'(x)}{2\sqrt{f(x) + g(x)}}$$

$$y'(1) = \frac{f'(1) + g'(1)}{2\sqrt{f(1) + g(1)}} = \frac{3 - 2}{2\sqrt{4 + 2}} = \boxed{\frac{1}{2\sqrt{6}}}$$

$$y = f(g(x)) \Rightarrow y' = f'(g(x)) \cdot g'(x)$$

$$y'(1) = f'(g(1)) \cdot g'(1) = f'(2)(-2) = 2(-2) = \boxed{-4}$$

Hit the Slopes – 400

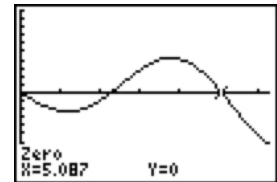
What is A?

$$y' = x(-\sin x) + \cos x$$

$$y'' = x(-\cos x) - \sin x - \sin x = -x \cos x - 2 \sin x$$

The maximum slope must occur where $y'' = 0$ and y'' switches from positive to negative. This is at 5.087. $y(5.087) = 1.861$.

Note : $y'(0) = 1, y'(2\pi) = 1$



Hit the Slopes – 500

What is D?

$$\frac{dy}{dx} = \frac{-2x}{e^{2y}} \Rightarrow e^{2y} dy = -2x dx$$

$$\frac{1}{2} e^{2y} = -x^2 + C$$

$$\frac{1}{2} = C \Rightarrow \frac{1}{2} e^{2y} = \frac{1}{2} - x^2 \Rightarrow e^{2y} = 1 - 2x^2$$

$$2y = \ln(1 - 2x^2) \Rightarrow y = \frac{\ln(1 - 2x^2)}{2}$$

$$\text{Domain : } 1 - 2x^2 > 0 \Rightarrow 2x^2 < 1 \Rightarrow x^2 < \frac{1}{2} \Rightarrow \boxed{-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}}$$