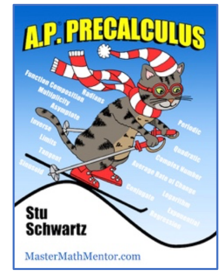
Topic 1.10 – Polynomial and Rational Inequalities – Classwork



In algebra, we followed up our study of solving linear equations with solving linear inequalities. It was straightforward. Mostly it ended up being a switch of signs. If the solution to $4x-10=2x+8$ is x = 9, then the solution to $4x-10>2x+8$ is x > 9. The only caveat is multiplying or diving both sides of an inequality by a negative number. In that case, we had to change the direction of the inequality sign. So if $-4x \le 12$, then dividing both sides by –4, we get $x \ge -3$. Of course, we could get around that rule by putting the variable x to the side of the inequality where it would have a positive coefficient.

To check an equation, you simply plug in the solution and show both sides were equal. For $4x-10=2x+8$, our check shows that 4(9) – 10 = 2(9) + 8 or 26 = 26.

But for an inequality, our answer is an interval so you plug in a number in that interval. For instance, for $4x-10>2x+8$, we plug in a number greater than 9. Using 10, we get 4(10) – 10 > 2(10) + 8 or 30 > 28.

Compound inequalities presents more work but no real issue. If $2x+3 \le 5x-9 < 16$, we create two inequalities: $5x-9 \ge 2x+3$ or $x \ge 4$ and $5x-9<16$ or $x<5$. Our final solution is $4 \le x < 5$ or $\left[4,5\right)$.

However, once we move to quadratic inequalities all bets are off. For instance, consider $x^2-2x>8$. Using our method to solve equations, we put all terms on one side and factor. $(x-4)(x+2)>0$. At this point, some logic must be applied. For 2 numbers multiplied together to be positive either both must be positive or both must be negative. Solving polynomial inequalities in this way involve examining many different cases.

However, there is a much easier way to solve this inequality. We pretend it's an equation and find the zeros of each factor. At this point, we create a number line, placing the numbers below the line, creating 3 intervals.

$$\underset{-2\qquad\qquad 4}{\rule{6cm}{0.4pt}}$$

. We test sample numbers in those 3 intervals, finding the sign of $(x-4)(x+2)$. No arithmetic is needed, just the sign. We get this result.

$$\overset{+\qquad\qquad -\qquad\qquad +}{\underset{-2\qquad\qquad 4}{\rule{6cm}{0.4pt}}}$$

. Since we are solving $(x-4)(x+2)>0$, our answer is x > 4 or x < –2. This process, as we said before, is called *interval work.*

Here are guidelines to solve inequalities of polynomials and rational function: They are similar to the methods we used to graph them.

- Place all terms on one side of the inequality and 0 on the other.
- Place the expression in fraction form. This might mean getting a common denominator.
- Factor both number and denominator.
- We find *critical values*, values of x where the expression either equals zero or is undefined. This involves setting the factors of numerator and denominator equal to zero.
- Do interval work with the critical values. If the multiplicity of any critical value is even, then the sign does not change about that critical value. If the multiplicity of all critical values are odd, then the signs alternate. That means you can do one of them and quickly fill the remainder in.
- Examining the original inequality, express your answer using intervals. If the inequality is a ≥ or ≤, then include the zeros of the numerator but do not include the zeros of the denominator in your interval notation.

Solve the following inequalities. Confirm graphically.

1) $9-x^2>0$

There are 2 ways to confirm graphically.

- Graph Y1 = $9-x^2$
  You want intervals where Y1 is above x-axis.
- Graph Y1 = $\left(9-x^2\right)\geq 0$
  You want intervals where Y1 is true (equal to 1)

2) $2x^2+3\geq 7x$

3) $x^3+64\leq 4x^2+16x$

4) $\dfrac{x^3}{4}>x^2+3x$

5) $\dfrac{x-4}{x+2}>0$

6) $\dfrac{x^2}{x-5}\leq 0$

7) $\dfrac{x-1}{-2x^2+50}\geq 0$

8) $\dfrac{x}{x^2-x-30}\leq 0$

9) $\dfrac{x^3}{x^4-16}\leq 0$

10) $\dfrac{8}{x^6}>\dfrac{1}{x^3}$

Topic 1.10 – Polynomial and Rational Inequalities – Homework

Solve the following inequalities. Confirm graphically.

1. $x^2 - 7x > 8$

2. $6x^2 < x + 2$

3. $x^4 \le x^3$

4. $4x^4 < 25x^2$

5. $x^3 - 54 \le 9x - 6x^2$

6) $x^3 + \frac{x}{4} \le x^2$

7. $\frac{x+5}{x+4} \ge 0$

8. $\frac{-6x}{x^2 - 12x + 36} < 0$

9. $\frac{x^2 + 2x + 1}{-2x^2 + 98} \ge 0$

10. $\frac{x-1}{x^3 - x^2 - 20x} \le 0$

11. $\frac{x^3 + 1}{x^4 - 4x^2 + 4} < 0$

12. $\frac{3}{x-4} \ge -x$

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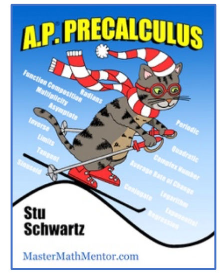
Topic 1.11 – Transformations – Classwork



Given a simple parent function $y = f(x)$ and its graph, certain modifications of the algebraic expression making up the function will result in predictable changes to the graph. The shape will essentially be the same but the graph can be moved up or down, left or right. It can get narrower or wider. It can compress or elongate. It can reflect across either the x or y-axis. The domain and/or range can change as well. Our goal is to be able detect and describe these modifications. We call this process transforming the graph.

Transformations $y = f(x), a > 0$ Sketching the new curve is best done by working with zeros.

Vertical Translations (Moving the curve up or down. Zeros are changed.)

$f(x)+a$ vertical shift up - translates the graph a units upwards

$f(x)-a$ vertical shift down - translates the graph a units downwards

Horizontal Translations (Moving the curve right or left. Zeros move *a* units right or left.)

$f(x-a)$ horizontal shift right - translates the graph a units to the right

$f(x+a)$ horizontal shift left - translates the graph a units to the left

Dilations (Making the curve narrower or wider, compressed or elongated)

$a \cdot f(x)$ Vertical stretch - if $a > 1$, the graph is narrower, if $a < 1$, it is wider.
Zeros don't change.

$f(ax)$ Horizontal stretch – if $a > 1$, the graph compresses, if $a < 1$, it elongates.
Compressing and elongating can have the effect of narrowing and widening
Zeros change, either compressing or elongating by a factor of a.

Reflections (Obtaining a mirror image of the curve)

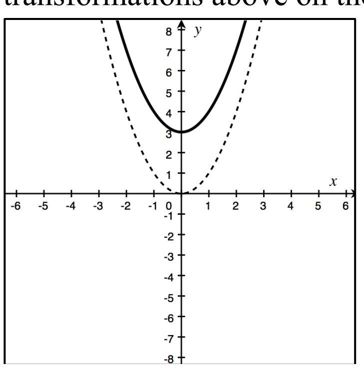
$-f(x)$ Reflection - flips the graph across the x-axis. Zeros don't change.

$f(-x)$ Reflection – flips the graph across the y-axis. Zeros change

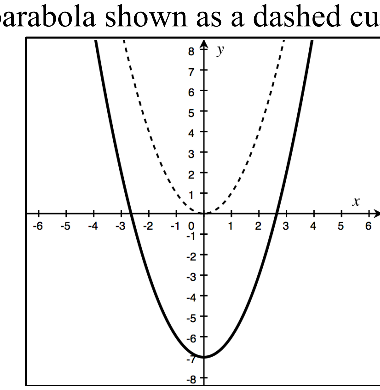
$|f(x)|$ Anything below the x-axis reflects across the x-axis. Zeros don't change.

$f|x|$ Anything to the right of the y-axis reflects across the y-axis. Some zeros change.

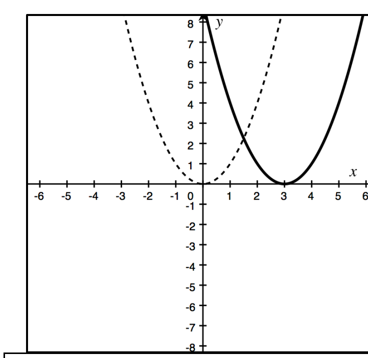
To start with, let $f(x) = x^2$. We know its domain is $(-\infty, \infty)$ and its range is $[0, \infty)$. Let's show some of the transformations above on the resulting parabola shown as a dashed curve.



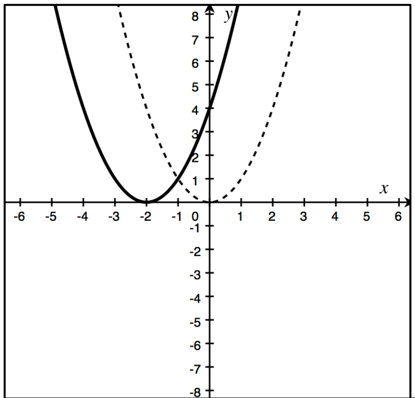
$f(x)+3 = x^2+3$
Vert trans: shifted 3 up
Domain: $(-\infty,\infty)$, Range: $[3,\infty)$



$f(x)-7 = x^2-7$
Vert trans: shifted 7 down
Domain: $(-\infty,\infty)$, Range: $[-7,\infty)$



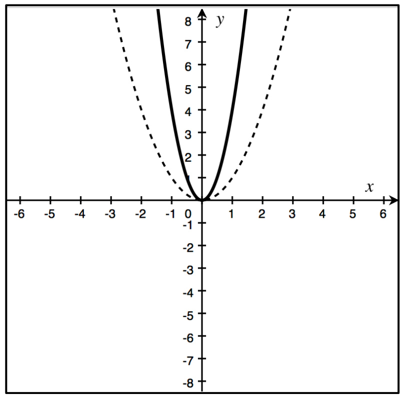
$f(x-3) = (x-3)^2$
Horiz trans: shifted 3 to the right
Domain $(-\infty,\infty)$, Range: $[0,\infty)$



$f(x+2)=(x+2)^2$

Horiz trans: shifted 2 to the left

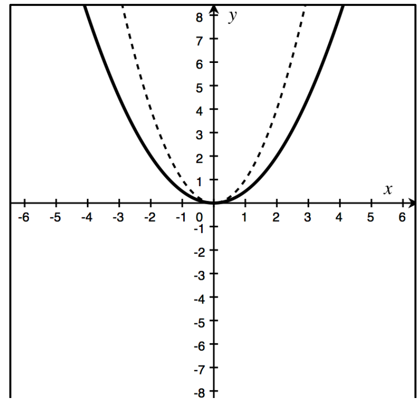
Domain: $(-\infty,\infty)$, Range: $[0,\infty)$



$4f(x)=4x^2$

Dilation: graph is narrower

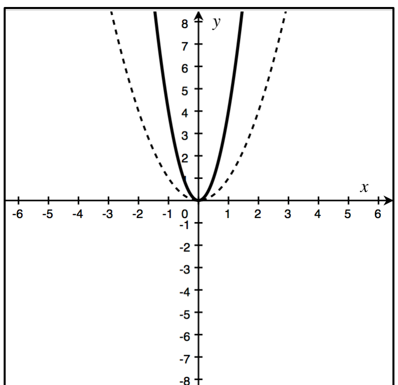
Domain: $(-\infty,\infty)$, Range: $[0,\infty)$



$0.5f(x)=0.5x^2$

Dilation: graph is wider

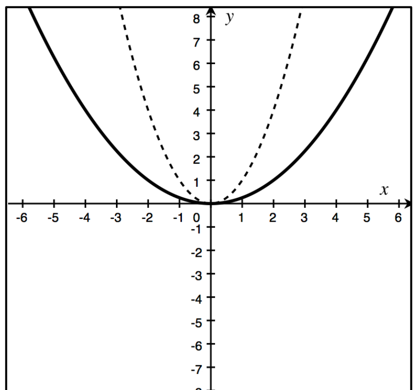
Domain: $(-\infty,\infty)$, Range: $[0,\infty)$



$f(2x)=(2x)^2=4x^2$

Dilation: graph compresses

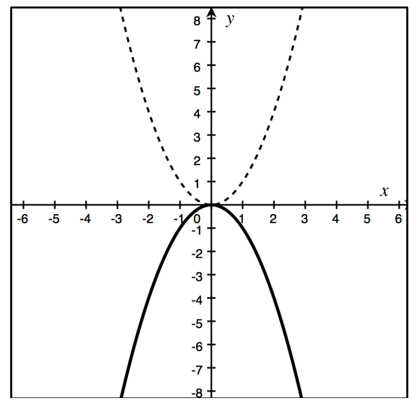
Domain: $(-\infty,\infty)$, Range: $[0,\infty)$



$f(0.5x)=(0.5x)^2=0.25x^2$

Dilation: graph elongates

Domain: $(-\infty,\infty)$, Range: $[0,\infty)$

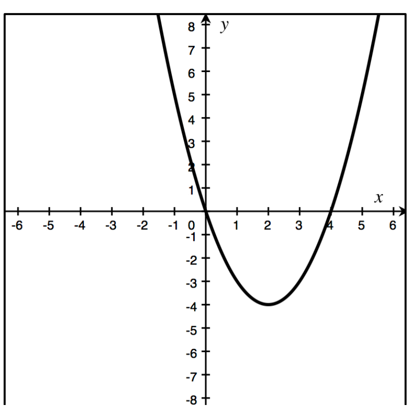


$-f(x)=-x^2$

Reflection: flips across $x-$axis

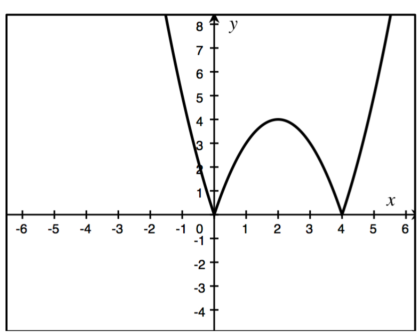
Domain: $(-\infty,\infty)$, Range: $(-\infty,0]$

Let's look at two dilations we haven't investigated yet. To do that, we will use $f(x)=x^2-4x$.



$f(x)=x^2-4x$

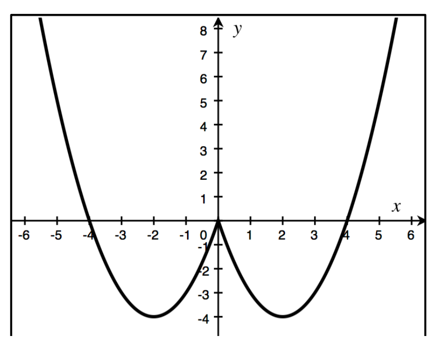
Domain: $(-\infty,\infty)$, Range: $[-4,\infty)$



$|f(x)|=|x^2-4x|$

Reflection: negative y becomes positive

Domain: $(-\infty,\infty)$, Range: $[0,\infty)$



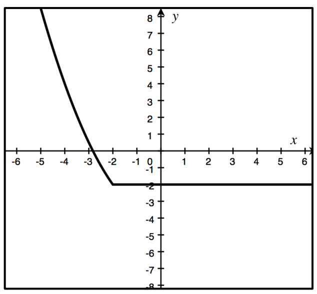
$f(|x|)=|x|^2-4|x|=x^2-4|x|$

Reflection: positive x across y-axis

Domain: $(-\infty,\infty)$, Range: $[-4,\infty)$

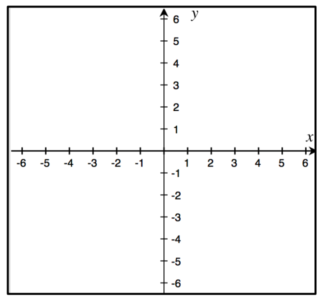
Examples) Given is a graph of $f(x)$. Sketch the transformation and find its domain and range.

1)



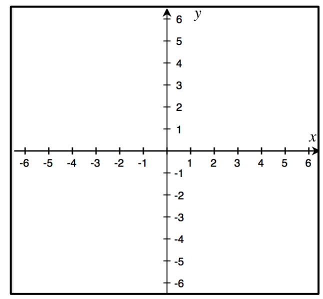
$f(x)$

Dom: Ran:



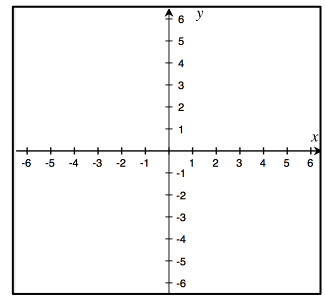
$f(x)+3$

Dom: Ran:



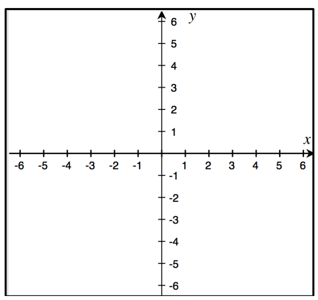
$f(x)-1$

Dom: Ran:



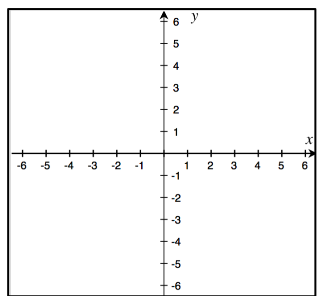
$f(x-2)$

Dom: Ran:



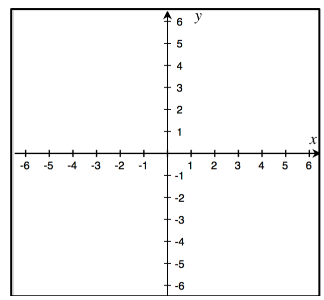
$f(x+1)$

Dom: Ran:



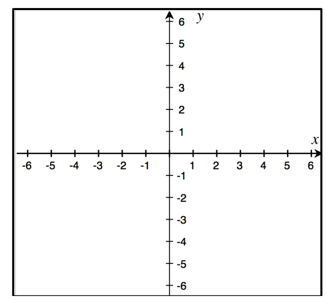
$2f(x)$

Dom: Ran:



$-f(x)$

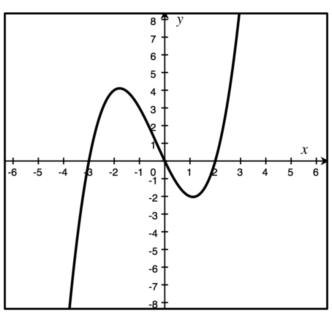
Dom: Ran:



$|f(x)|$

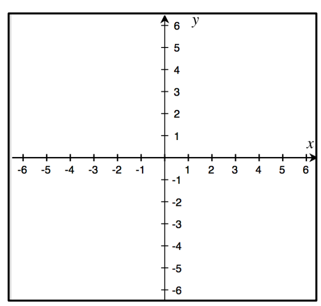
Dom: Ran:

2)



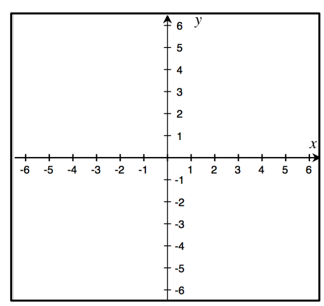
$f(x)$

Dom: Ran:



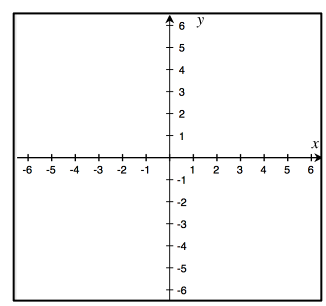
$f(x-3)$

Dom: Ran:



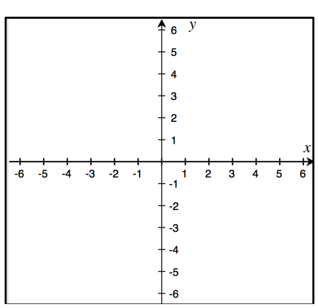
$f(x+2)$

Dom: Ran:



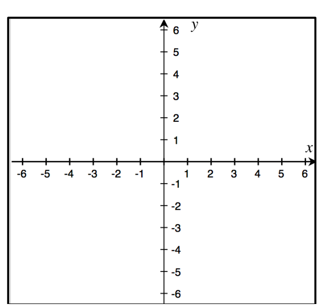
$f(2x)$

Dom: Ran:



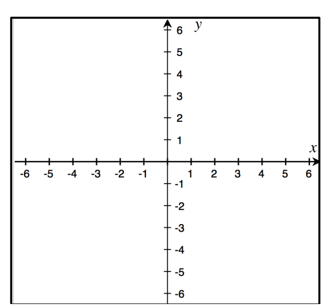
$f(0.5x)$

Dom: Ran:



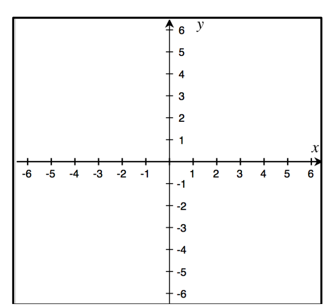
$f(-x)$

Dom: Ran:



$|f(x)|$

Dom: Ran:



$f(|x|)$

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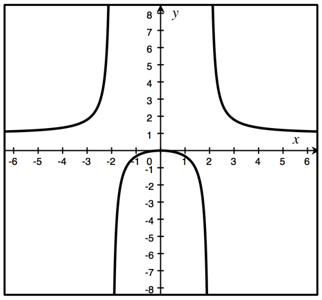
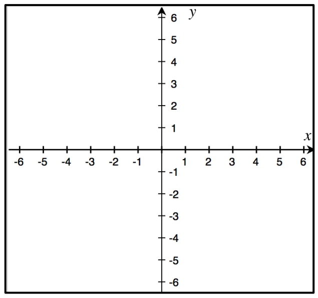
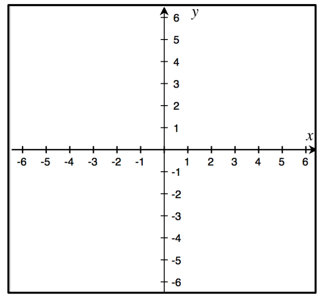
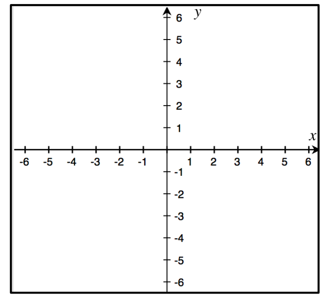
3)    

$f(x)$	$f(x)-3$	$f(x+2)$	$f(2x)$
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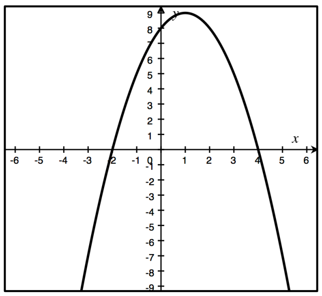
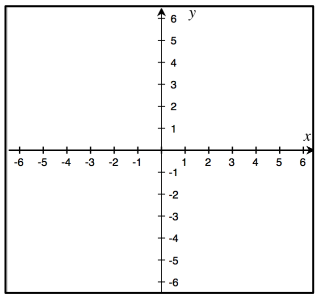
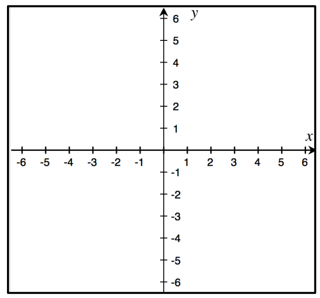
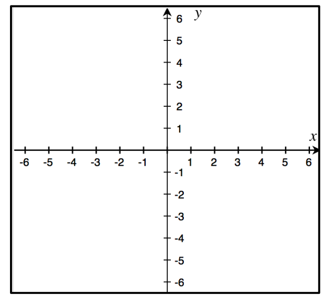
4)    

$f(x)$	$f(x-1)-3$	$4-f(x)$	$\lvert f(x)\rvert-3$
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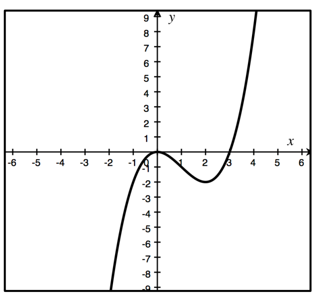
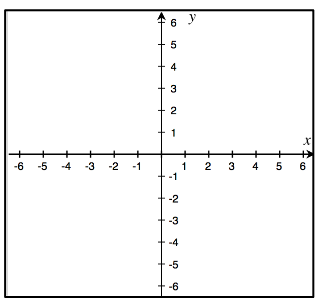
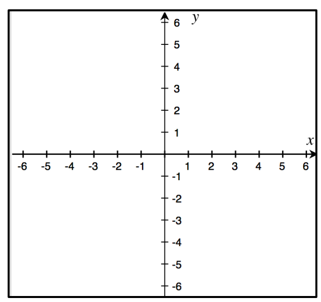
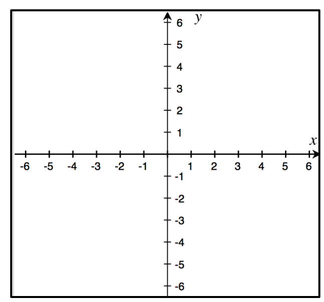
5)    

$f(x)$	$2f(x+2)$	$-f(0.5x)$	$4f\lvert x\rvert$
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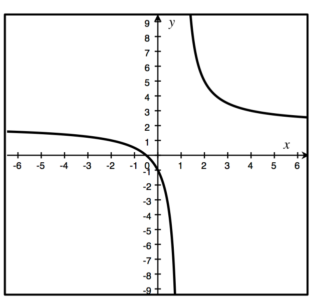
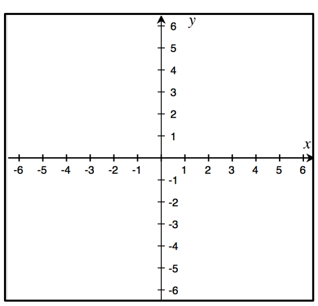
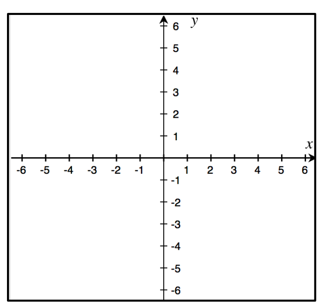
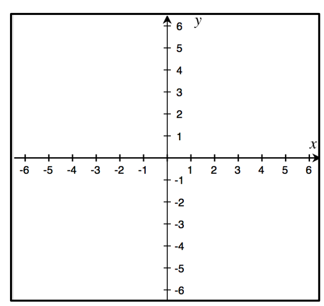
6)    

$f(x)$	$0.5f(x+2)$	$-2f(2x)$	$\lvert f(x)-2\rvert$
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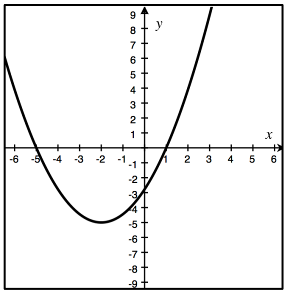
Topic 1.11 – Transformations – Homework

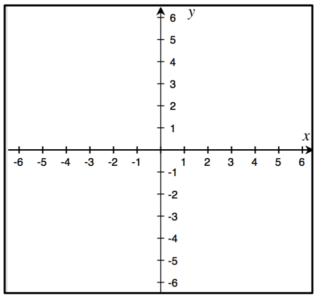
Given is a graph of $f(x)$. Sketch the transformation and find its domain and range.

1.



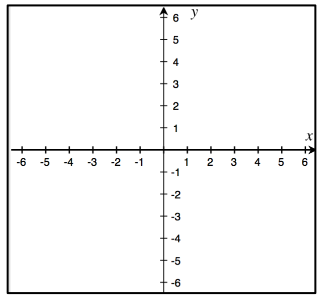
$f(x)$

Dom: Ran:



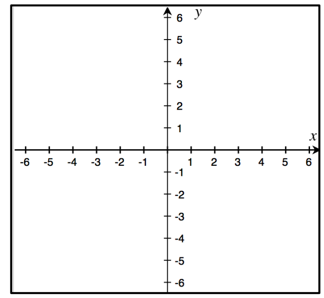
$f(x)+5$

Dom: Ran:



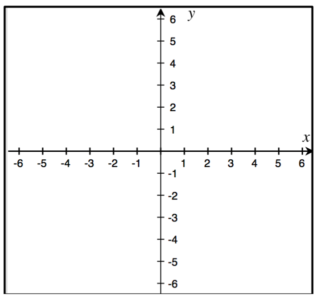
$f(x)-4$

Dom: Ran:



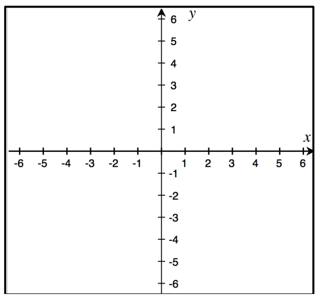
$f(x+1)$

Dom: Ran:



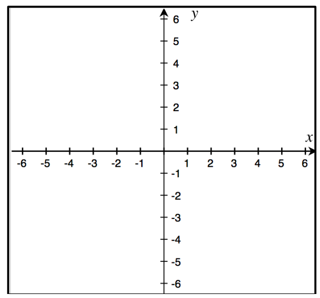
$f(x-3)$

Dom: Ran:



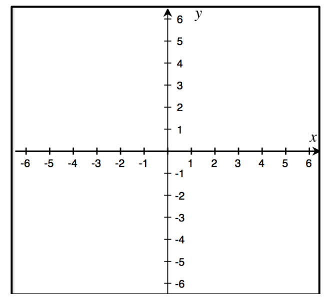
$1.5f(x)$

Dom: Ran:



$0.5f(x)$

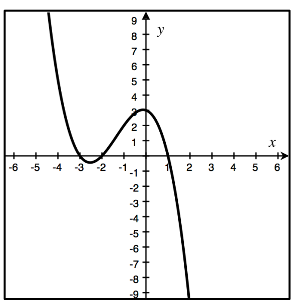
Dom: Ran:



$-f(x)$

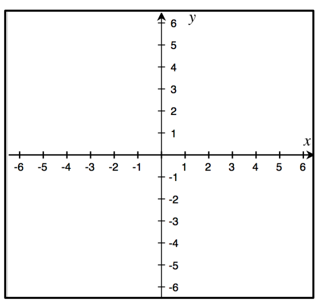
Dom: Ran:

2.



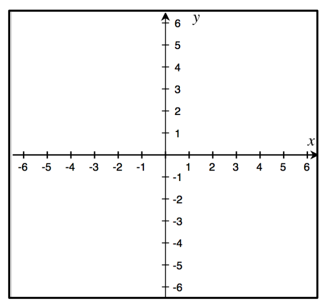
$f(x)$

Dom: Ran:



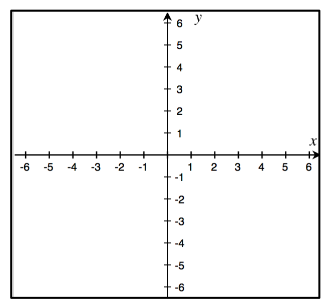
$2f(x)$

Dom: Ran:



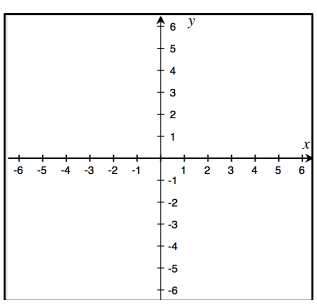
$f(2x)$

Dom: Ran:



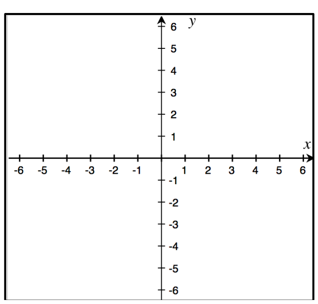
$0.5f(x)$

Dom: Ran:



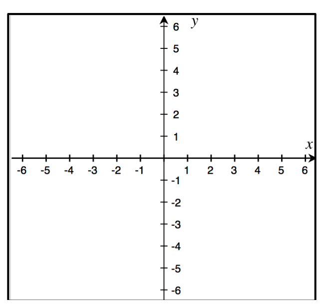
$f(0.5x)$

Dom: Ran:



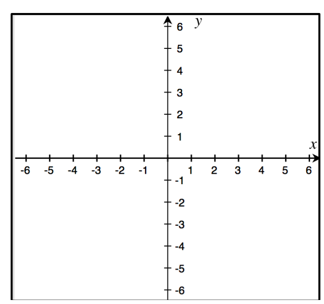
$-f(x)$

Dom: Ran:



$|f(x)|$

Dom: Ran:



$f(|x|)$

Dom: Ran:

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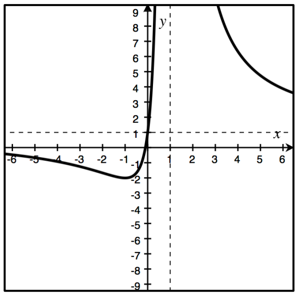
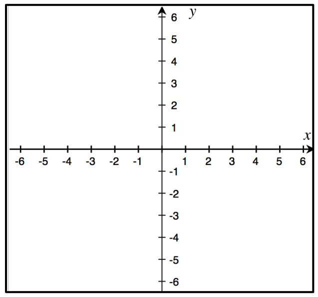
3.



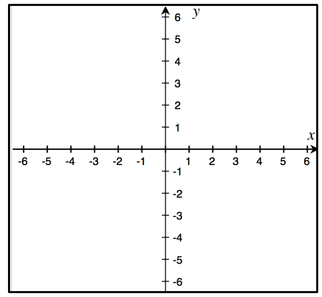
$f(x)$

Dom: Ran:



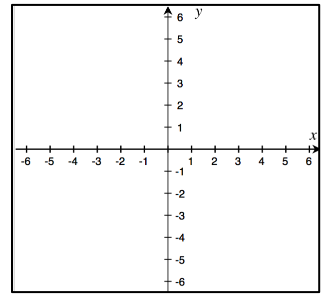
$f(x)-3$

Dom: Ran:



$f(x-2)$

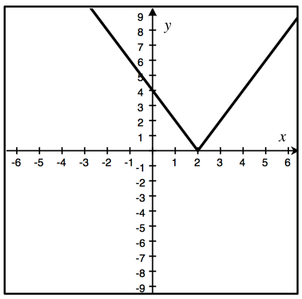
Dom: Ran:



$f(-x)$

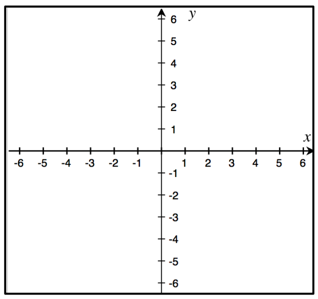
Dom: Ran:

4.



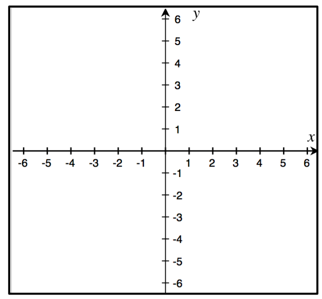
$f(x)$

Dom: Ran:



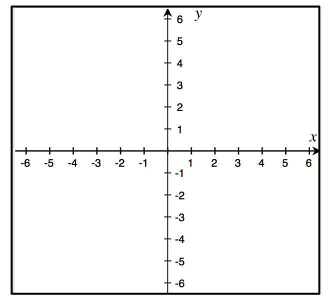
$f(x-2)-2$

Dom: Ran:



$-f(x+3)$

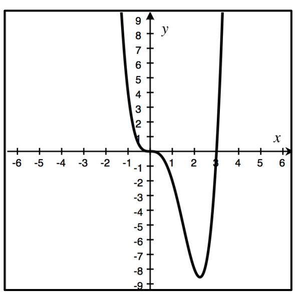
Dom: Ran:



$f(2x)+2$

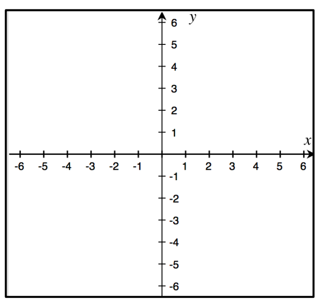
Dom: Ran:

5.



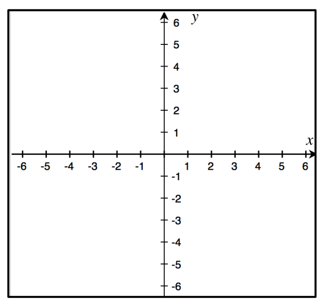
$f(x)$

Dom: Ran:



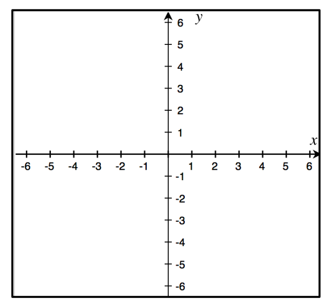
$0.5f(0.5x)$

Dom: Ran:



$-f(-x)$

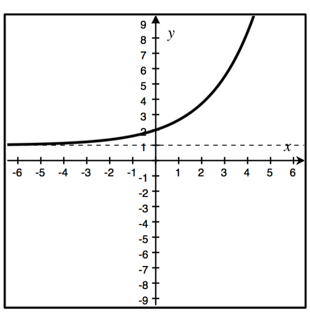
Dom: Ran:



$\left|f(|x|)\right|$

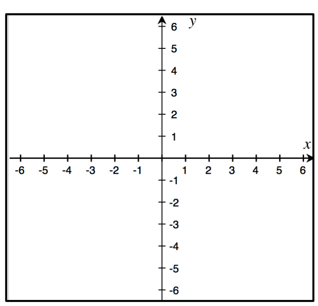
Dom: Ran:

6.



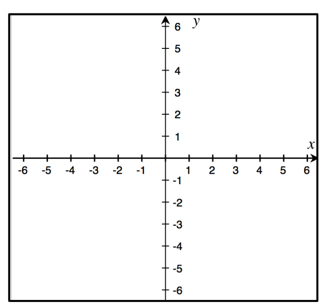
$f(x)$

Dom: Ran:



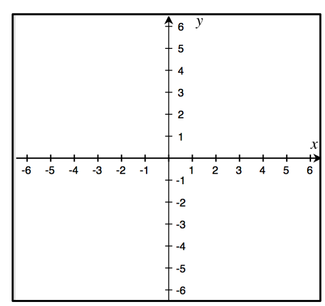
$f(x-3)-3$

Dom: Ran:



$2f(2x)$

Dom: Ran:



$-0.5f(|x|)$

Dom: Ran:

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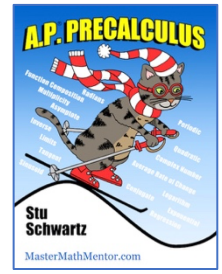
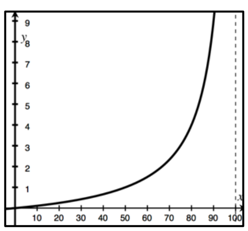
Topic 1.12 – Rational Function Modeling – Classwork



In section 1.7, we focused on polynomial modeling – that is modeling real-world situations whose solution was based on a polynomial. In this section, the equation generated will be a rational function. However all these techniques that were recommended with polynomial modeling are in play here in addition to understanding the real-life role of asymptotes. There are many examples of asymptotic behavior in real-life using inverse proportionality.

Example) In Florida, a major hurricane like Hurricane Ian devastated the area where it came onshore and subsequently its eventual path. Many wooden power poles were destroyed and power could not be restored until they were replaced. The cost of putting up concrete poles that are impervious to winds is not a linear function. That is if it cost C dollars to replace 20% of the poles, it would cost more than $2C$ to replace 40% of the poles. That is because many poles are in remote sections with difficult access. The greater the value of percent p, the faster C increases. The cost of removing p% of the wooden poles is given by, $C = \dfrac{p}{100-p}, 0 \le p < 100$, where C is measured in tens of millions of dollars.

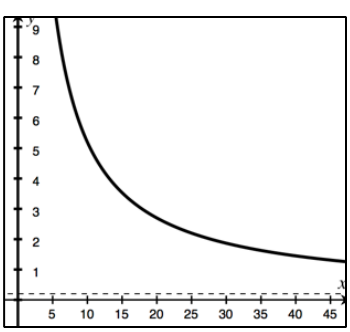
a) If Florida currently has removed 30% of the wooden poles, having replaced them with concrete, how much more would it cost to remove 40% of the poles?
b) Down the road, Florida hopes to go from 50% of the poles removed to 60%. What is the cost?
c) What is the vertical asymptote and what does it represent in the context of the problem?



a. $C(40) - C(30) = 0.666,667 - 0.428571 = 0.238095(10) \approx 2.381$ million
b. $C(60) - C(50) = 1.5 - 1 = 0.5(10) \approx \$5$ million
c. $p = 100$. As the percentage gets closer to 100, the cost becomes prohibitive.

Example) When a cruise ship comes into port, a major job is to clean all of its windows. With a ship being as much as 15 decks high, this takes a great deal of time and based on other considerations, many staff are involved in the project. The amount of time T that it takes to clean all the windows can be modeled by $T = \dfrac{s+250}{5s}, 0 \le s \le 50$, where s is the number of staff members cleaning the windows.

a) Find the amount of time it would take to clean all windows if 25 staff are allotted to the project.
b) The captain assigns 35 staff but 5 of them are sick. What is the increase in time?
c) The ship will only be in port 5 hours. How many staff should be allotted to the project?
d) What is the horizontal asympote and what does it represent?



a. $T(25) = \dfrac{25+250}{5(25)} = \dfrac{300}{125} = 2.2$ hours

b $T(30) - T(35) = \dfrac{30+250}{5(30)} - \dfrac{35+250}{5(35)} = 0.238$ hours or about 14 minutes

c. $\dfrac{s+250}{5s} = 5 \Rightarrow 24s = 250 \Rightarrow s = \dfrac{250}{24} \approx 10$ staff

d. $\lim\limits_{s\to\infty} \dfrac{s+250}{5s} = \dfrac{1}{5}$ hour = 12 minutes - the minimum time it could take with infinite staff

Realize that the domain of this function is positive integers and the graph is a series of points rather than a continuous curve. If the problem was amended to represent the percentage of effort the washers put into the cleaning, then a curve makes sense.

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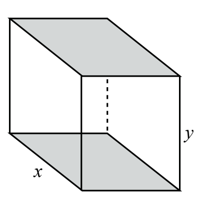
1) Find two positive numbers a and b that first minimizes and then maximizes the sum of twice a and b if the product of the two numbers is 288. Complete the table.

a	1	288	2	144	4	a
b						b
Sum						

2) The intensity of a light bulb varies inversely as the square of the distance from the bulb. The function $I(d) = \dfrac{k}{d^2}$ models this situation where I is the light intensity, d is the disance from the light source, and k is a constant. Suppose the light intensity 6 feet away from a light bulb is 20 lumens. Graph the function. a) The intensity of the bulb is what percent dimmer at 3 compared to 2 feet. b) What is the role of the horizontal asympote in this problem?

3) The size of a population of bacteria (measured in thousands) introduced to a food grows according to the formula $P(t) = \dfrac{6000t}{60 + t^2}$ where t is measured in days. a) Determine what day the bacteria population is at a maximum and find the maximum size of the population. b) Determine the day(s) the population is 300,000. c) Explain the role of an asymptote in this problem.

4) 

A closed box with a square base is to have a volume of 1,800 in^3. The material for the top and bottom of the box (the shaded section in the figure) costs 30 cents per square inch while the material for the sides costs 10 cents per square inch. a) Find the minimum cost of the box. b) What is the role of the vertical asymptote in this problem?

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5) A rectangular area is to be fenced in using two types of fencing. The front and back use fencing costing \$5 a foot while the sides use fencing costing \$4 a foot. If the area of the rectangle must contain 500 ft^2, what should the dimensions of the rectangle be in order to keep the cost a minimum?

6) The state gaming commission introduces trout into a lake. The population P of trout in the lake is modeled by $P(t)=\dfrac{25(4t+7)}{1+0.08t}$ where t is measured in weeks.
a) How much will the trout population increase in one year? b) how long will it take to double? c) What is the limiting size of the trout population? d) How long will it take for the trout population to grow from 90% to 95% of its maximum size?

7) Alligator Alley is a road that connects Ft. Lauderdale to Naples, Florida. The distance is 120 miles. A salesman averages 60 mph for the roundtrip. Let x be the average speed outbound and y be the average speed inbound, both in miles per hour. a) show that $y=\dfrac{30x}{x-30}$. b) Determine the asympotes and explain their significance.

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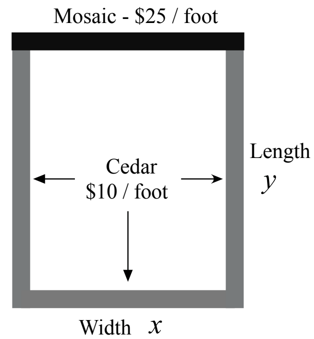
Topic 1.12 – Rational Function Modeling – Homework

1. The owner of a nursery wants to add a 1,000 ft^2 rectangular area to its greenhouse to sell seedlings. For aesthetic reasons, he has decided to border the area on three sides by cedar siding at a cost of \$10 per foot, as shown by the figure to the right. The remaining side is to be a wall with a brick mosaic that costs \$25 per foot. Complete the table. a. What should the dimensions of the sides so that the cost of the project will be minimized? b. What is that cost? c. Explain the role of the asymptote in the context of the problem.



Width	Length	Cedar Cost	Mosaic Cost	Total Cost
10				
100				
1000				
x	y			

2. An electronics company estimates that the cost in dollars of producing x units of a new model TV is given by $C(x) = 80000 + 40x + 0.2x^2$. Find the production level that minimizes the average cost per unit. Complete the table. Explain why the average cost to produce TV's is so great for either small or large number of TV's.

Units	Cost	Average cost
10		
100		
1,000		
2,000		
5,000		

3. A moving company calculates charges for delivery based on the average velocity v the truck travels as well as the time it takes:

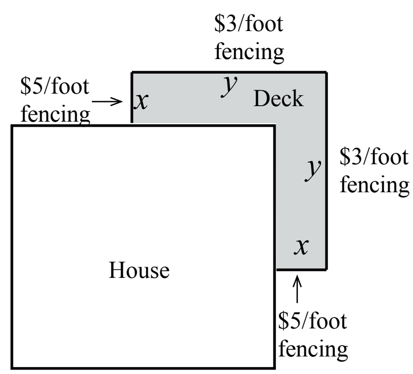
Fuel cost: $\dfrac{v^2}{60}$ per hour Driver cost: \$60 per hour

a. Complete the table. b. Find the velocity v that the truck should travel in order to minimize the cost of a 120-mile trip. c. What is that cost? d. Explain the role of the asymptote in the context of the problem.

v	10	20	40
time t			
Fuel cost/hr			
Fuel cost			
Driver cost			
Total			

4. Mr. Tyree is building a symmetric deck of constant width on the corner of his house as shown in the figure to the right. Fencing is placed on the edges of the deck with no fence needed against the house. Fencing perpendicular to the house costs \$5/foot while fencing parallel to the house costs \$3/foot.

If Mr. Tyree wants a deck with area 200 ft^2, what is the minimum cost to build the deck?



Note: This problem might look familiar. We had a similar problem in section 1.7, number 8. That problem had a constraint that Mr. Tyree has a certain budget for the deck and wishes to build the largest area deck he could. This problem has a constraint that the deck must be a certain size he wants to build the deck as cheaply as he can. This is a typical conundrum every day. People want a new vehicle with all the new bells and whistles and yet want to purchase it as cheaply as possible. Rarely do those two circumstances occur simultaneously.

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5. The concentration C of a medication into the bloodstream is a ratio of the mass of the medication and the volume of the solution. A simple blood test can be used to determine this. The greater the value of C, the greater the concentration. The concentration of Flomax into the bloodstream t hours after taking a pill is given by $C(t)=\dfrac{4t^2+3t}{t^3+45}, t>0$. A. When is the greatest concentration of Flomax into the body. b. When approxmately is the concentration in the bloodstream increasing the fastest? c. Approxmately when is the concentration in the bloodstream falling the fastest? d. About how long will it take for the concentration of Flomax in the bloodstream to be considered "just a trace?" ($C < 0.05$).

6. A condo association board votes to contact its residents electronically, rather than by mail, saving money. However, forms must be prepared and residents must sign off in order for this to occur. The number of days N to get responses from p% of its residents is given by $N(p)=\dfrac{5p+1000}{100-p}$. This includes the time to prepare and mail out the forms. a. How long did it take to get the first 20% of the residents to respond? b. After a month (30 days), the board is not satisfied with the lack of response so they send out another letter. What percentage of the residents responded in the next month? c. What is the vertical asymptote and its meaning in context?

7. Airline flight time is measured from gate to gate. The scheduled flight time from Charlotte, North Carolina to New York City is 2 hours. The plane's first leg takes it to Washington, DC a distance of 400 miles and the second leg into New York City, a distance of 200 miles. If x is the speed of the first leg in mph and y the speed of the second leg, a. show that $y=\dfrac{100x}{x-200}$. bc Determine the asympotes and their significance.

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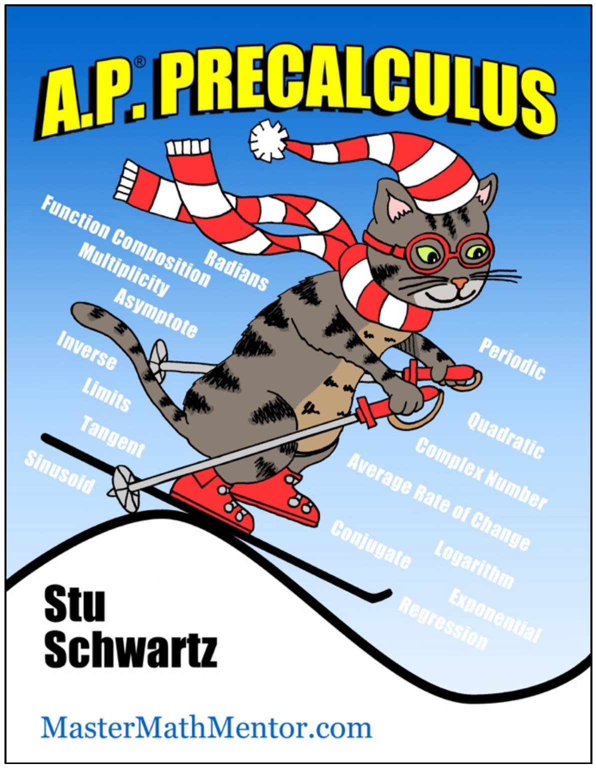
Unit 2
Exponential and Logarithmic Functions



#	Unit 2 Topic	Class-Work	Home-Work
1	Arithmetic Sequences	2-1	2-5
2	Geometric Sequences	2-7	2-10
3	Exponential Functions	2-12	2-18
4	Composition of Functions	2-21	2-24
5	Inverses	2-27	2-31
6	Logarithms	2-32	2-38
7	Property of Logarithms	2-40	2-45
8	Exponential Models	2-49	2-60
9	Regression	2-65	2-73
10	Semi-Log Plots	2-77	2-79