

# A.P.<sup>®</sup> PRECALCULUS



Function Composition  
Multiplicity

Radians

Asymptote

Inverse

Limits

Tangent

Sinusoid

Periodic

Quadratic

Complex Number

Average Rate of Change

Conjugate

Logarithm

Exponential  
Regression

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This manual was developed by Stu Schwartz for the new Advanced Placement<sup>®</sup> Precalculus course that begins in the Fall of 2023. The student manual is available for download from the [mastermathmentor.com](http://mastermathmentor.com) website free of charge and may be copied for purposes of face-to-face instruction. The student manuals and solution manuals may not be altered in any way in the copy process. Placing either the student version or solution version on the Internet or on school websites is illegal without special permission. Teachers may make printed copies of solutions but may not provide the solutions electronically except without special permission.

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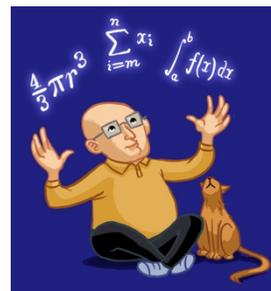
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Thanks to my friend and business partner Ted Tyree for his unwavering support and creativity in all my projects and for always finding time to bring my materials free to anyone who wants them through the Internet. Thanks to Sam Tsui (Yale University, 2011) for developing the [www.mastermathmentor.com](http://www.mastermathmentor.com) logo below and to Jason Nocera of Niche Cartoons <http://www.nichecartoons.com> for developing the cover art starring Fanucci the Cat. Thanks to Jen Castor of Wissahickon High School for her advice throughout the rewrite process and to Dr. Sheldon Goldberg for editing the work. Finally, thanks to all the teachers who are members of MasterMathMentor and who encouraged me to create the manual.

In a project of this nature, it is extremely possible that mathematical typos have worked their way into this manual. It is terribly difficult to find errors in the editing process. So, if you do find mistakes of any type please let me know at [team@mastermathmentor.com](mailto:team@mastermathmentor.com). I will immediately correct them, alert people through our errata page (giving you credit) and update the revision to our server.

As always, questions or comments are always welcome. Best regards,

Stu Schwartz  
March, 2023



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\* Topic not specifically covered on AP Precalculus Framework

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\* Topic not specifically covered on AP Precalculus Framework

## *To the Student*

Welcome to Advanced Placement Precalculus, the College Board's new mathematics course.

This course was primarily designed for students who will not be taking AP Calculus in high school. And yet, when you enter college, you very well may be entering a STEM (Science, Technology, Engineering, Mathematics) field or one that is on its periphery. In that case, some basic knowledge of calculus is needed. In the past, students enter a University level calculus course and get blown away because many of the required skills they need in this course is lacking. In that case, students have to take the college precalculus course first. Taking this AP precalculus course and doing well in the AP exam ensures that you will have all the skills needed in that first university calculus course which can be intimidating to students.

However, this course is intended to do double duty. You will study a broad spectrum of function types that are foundations for careers in biology, health science, social science and data science. Modeling, a central instructional theme for the course, helps students come to a deeper understanding of the analytical skills and why they are important. The types of problem situations encountered are no longer of the mundane types: “John is twice as old as Mary”, “How many cantaloupes did Edna buy?”, “How many nickels does Bob have?”, and “Find 3 consecutive numbers whose sum is 54.” Rather the problems are meaningful to real life whether it is finding the value of a monthly deposit after 5 years of gaining interest, estimating the amount of sunlight in a certain city on a certain day, or the cheapest cost to build a structure of a given size, to name a few. No longer will you be asking your math teacher “when are we ever going to use this?”

The structure of this manual matches the structure of my AP Calculus manuals that many teachers use worldwide. It is meant to be used in conjunction with your teacher. It is not a textbook that shows all the methodology of solving problems. Rather it is a workbook. There is a classwork section that gives your teacher his or her lesson plans. It covers every aspect of the topic that is required to be taught. There are examples that you will write in solutions as your teacher explains them. I believe that you learn math by writing it – and not just reading it. The better you complete all the examples with the teacher acting as guide, the better tool you will have for studying it later.

You then have the homework section which again, leaves space for you to write in your solutions. The problems in homework mirror that of what was shown in classwork.

The advantage of this manual is that if you are absent from class, you have the class notes. You ask the teacher what pages you missed and you either ask the teacher, or better yet, get help from a friend in catching you up. You are never having to write down the problem, just the solutions.

By the end of the course, you will have the entire manual filled in. When you encounter that first calculus college class, you can look at the manual to refresh your memory. Hundreds of students have told me how they still refer to their AP Calculus manual when they are taking advanced math courses that use calculus.

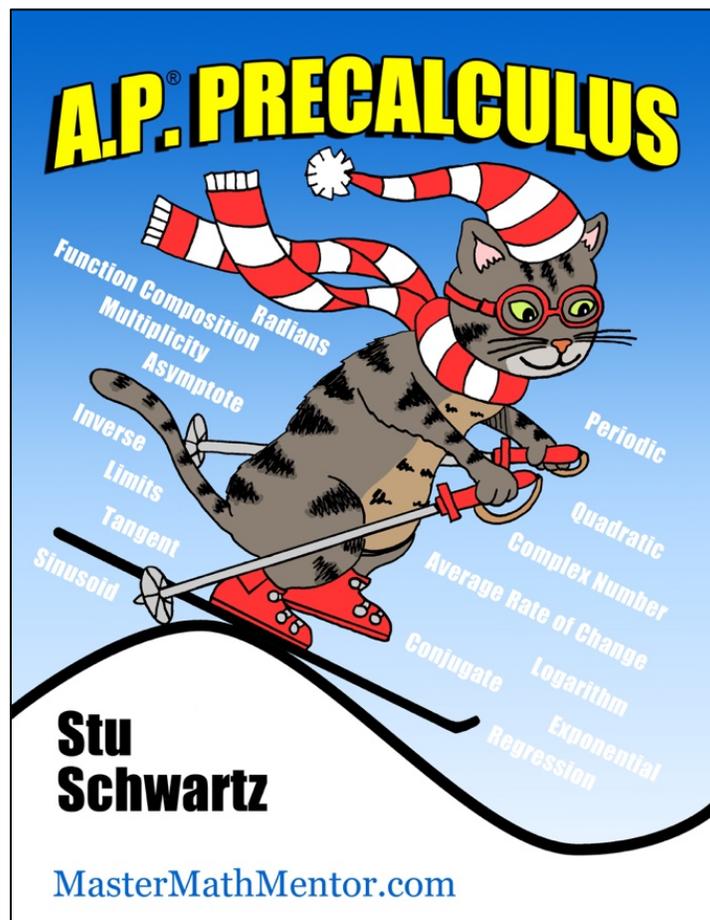
This manual was developed in 6 months. It is essentially a one-man operation and despite much proofreading, errors still creep in. So if you spot something incorrect, please contact me at [team@mastermathmentor.com](mailto:team@mastermathmentor.com). If you are right, I will correct the error. And if you have a question, just ask. I will respond personally.

My goal is not only to make it easy for you to learn AP precalculus but more importantly, for you to enjoy the course and get you excited where math can take you. I wish you the best of luck.

Yours mathematically,  
Stu Schwartz and Fanucci the Cat

# Unit 1

## Polynomials and Rational Functions



#	Unit 1 Topic	Class-Work	Home-Work
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3	Change in Linear and Quadratic Functions	1-15	1-19
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5	Polynomials and their Zeros	1-27	1-36
6	End Behavior	1-39	1-43
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8	Rational Functions	1-57	1-62
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11	Transformations	1-76	1-82
12	Rational Function Modeling	1-82	1-85